Technical Notes

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Spreading of Turbulent Mixing Layers

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Nomenclature

A = constant occurring in Eq. (4)
b = local length scale
f = self-preserving form of mean velocity $u = u_2+(u_1+u_2)f$
K_G and K_γ = constants in constant eddy viscosity assumptions
$\langle q^2\rangle$ = $\langle(u')^2\rangle+\langle(v')^2\rangle+\langle(w')^2\rangle$
u = component of mean velocity in x direction
u_1, u_2 = mean velocities of fast and slow streams, respectively
u', v', w' = fluctuating components of velocity
x, y = streamwise and transverse coordinates, respectively
δ = mixing layer width
ν_T = eddy viscosity
ζ = similarity variable, $\zeta = \sigma y/x$
σ = similarity parameter

GÖRTLER[1] derived analytical mean velocity distributions for two-dimensional, self-preserving mixing layers by assuming an eddy viscosity

$$\nu_T = K_G(u_1-u_2)\delta \tag{1}$$

Solutions for $f(\zeta)$ were derived for different values of u_2/u_1 where

$$\sigma = \tfrac{1}{2}\{K_G(d\delta/dx)(u_1-u_2)/(u_1+u_2)\}^{-\frac{1}{2}} \tag{2}$$

The shapes of experimental and analytical mean velocity profiles are generally in good agreement but experimental values of σ have a high degree of scatter (see Miles and Shih[2]). As will be shown σ is inversely proportional to the mixing layer spreading rate which is an important unknown in practical mixing layer flows. Miles and Shih assumed that the scatter in σ was due to separating plate boundary-layer effects and these boundary layers were removed by suction in their experiments. Their σ values lie on a smooth curve (Fig. 1).

There can be a self-preserving region in a mixing layer if the separating plate and working section wall boundary layers are thin but it is only by extensive turbulence measurements that the existence and position of this region can be established. Mean velocity measurements must not be used as a sole criterion for the existence of self-preservation hence the scatter in previous results.

Hot wire measurements[3,4] (Fig. 2) of the turbulence components and mean velocity have been made in mixing layers with $u_2/u_1 = 0.30$ and $u_2/u_1 = 0.61$. These measurements showed well-defined self-preserving regions and the mean velocity measurements agreed with Görtler's solution, the values of σ are included in Fig. 1. Other work which included turbulence measurements was that of Liepmann and Laufer[5] and Wygnanski and Fiedler[6] for the free mixing layer. The values of σ in Fig. 1 should be the most accurate available and apart from Wygnanski's value which is commented upon below they are within 5% of the empirical curve (for $0 \leq u_2/u_1 \leq 0.65$)

$$\sigma/\sigma_0 = (1+u_2/u_1)^{1/2}/(1-u_2/u_1) \tag{3}$$

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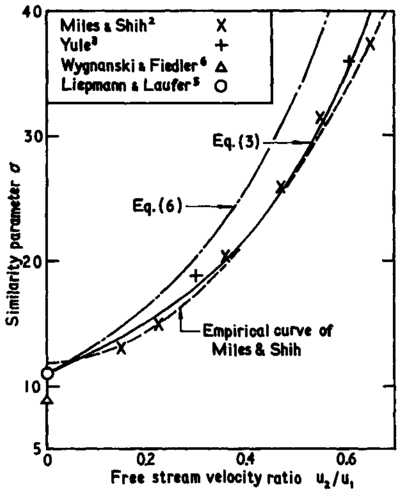

Fig. 1 Similarity parameter for self-preserving mixing.

where $\sigma_0 = 11$. Miles and Shih's values of σ for $u_2/u_1 > 0.65$ diverge from this curve and their empirical relation implies that σ is finite when $u_2/u_1 = 1$ which is not possible for a self-preserving flow. This divergence is not critical as measurements of the spreading rate and $f(\zeta)$ are liable to increasing inaccuracy as $u_2/u_1 \to 1$. It is proposed that Eq. (3) is an empirical relation for σ which satisfies experimental data and the physical boundary conditions.

Görtler based the eddy viscosity upon a length scale δ and a velocity scale (u_1-u_2) however δ cannot be used in practice because f has asymptotic boundary conditions and instead a local length scale b is introduced. If b is defined as the distance between two points (f_1, ζ_1) and (f_2, ζ_2) near the edges of the layer then the constant eddy-viscosity solution gives $(\zeta_1-\zeta_2)$ independent of u_2/u_1 to within 3% for $0.95 \geq f_1$ and $f_2 \geq 0.05$

$$\zeta_1-\zeta_2 = \sigma\, db/dx = \text{const} = A \tag{4}$$

In an analysis for the development region of a free shear layer Nash[7] based b upon the mean velocity gradient at the center of the mixing layer so that in the case of two stream mixing

$$b = -B\{\partial[(u-u_2)/(u_1-u_2)]/\partial y\}_{\max}^{-1} \tag{5}$$

Thus $db/dx = -B\sigma^{-1}(df/d\zeta)_{\max}^{-1}$ and as $(df/d\zeta)_{\max}$ is independent of u_2/u_1 in the constant eddy-viscosity solution this is identical to Eq. (4) with $B = -A(df/d\zeta)_{\max}$.

Two recognized methods of basing b upon the mean velocity distribution yield the same relation between the rate of spread and the similarity parameter. Equation (4) and the use of b instead of δ in Eq. (2) gives the relation

$$\sigma = (4K_G A)^{-1}(u_1+u_2)/(u_1-u_2) = \sigma_0(1+u_2/u_1)/(1-u_2/u_1) \tag{6}$$

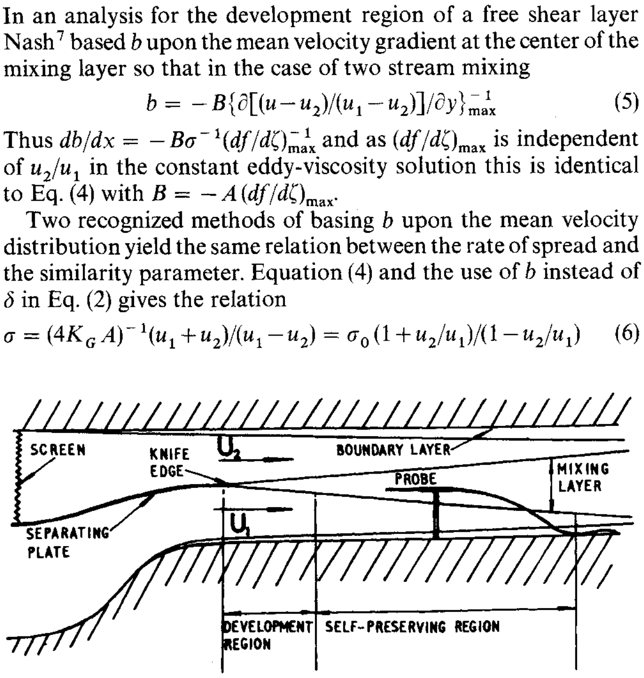

Fig. 2 Wind-tunnel installation for mixing layer investigation.

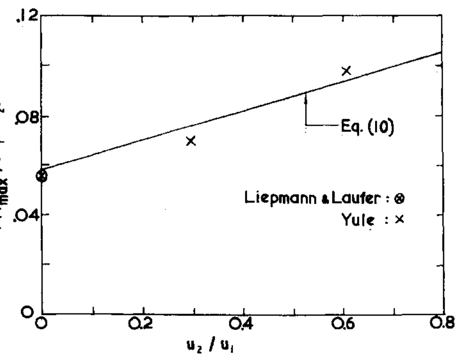

Fig. 3 Variation of dimensionless peak turbulence intensity with free-stream velocity ratio.

Agreement with experiment is poor (Fig. 1) however this discrepancy is explained when the eddy viscosity assumption is written in a more logical form. The hypothesis implies a dominating gradient diffusion process and such a process would be controlled by the energy containing turbulence. The eddy viscosity should be dependent upon this turbulence and not necessarily upon the local scales of the mean velocity distribution, thus a better choice of the velocity scale is the rms peak turbulence intensity $\langle q^2\rangle^{1/2}_{max}$. However it may be expected that the local turbulence dictates the width of the mixing layer and indeed measurements[3] show that $b = Ax/\sigma$, from Eq. (4), gives good correlation between the turbulence length scales at different velocity ratios. The new eddy-viscosity assumption is

$$\nu_T = K_\gamma b \langle q^2\rangle^{1/2}_{max} \tag{7}$$

and with this assumption σ is [c.f. Eq. (2)]

$$\sigma = \tfrac{1}{2}\{(K_\gamma db/dx)\langle q^2\rangle^{1/2}_{max}/(u_1+u_2)\}^{-1/2} \tag{8}$$

Equation (4) and Eq. (8) give

$$\sigma = (4K_\gamma A)^{-1}\{(u_1+u_2)/(u_1-u_2)\}\{\langle q^2\rangle^{1/2}_{max}/(u_1-u_2)\}^{-1} \tag{9}$$

The variation of $\langle q^2\rangle^{1/2}_{max}/(u_1-u_2)$ which is implied by the σ distribution is given by Eq. (3) and Eq. (9)

$$\langle q^2\rangle_{max}/(u_1-u_2)^2 = (4K_\gamma A\sigma_0)^{-2}(1+u_2/u_1) \tag{10}$$

Table 1 Experimental mixing layer parameters

Source	u_2/u_1	σ	$\langle q^2\rangle_{max}/(u_1-u_2)^2$	$K_\gamma A$	$K_G A$
Wygnanski and Fiedler[6]	0	9	0.071	0.104	0.028
Liepmann and Laufer[5]	0	11	0.056	0.096	0.023
Yule[3]	0.30	19	0.069	0.094	0.024
Yule[3]	0.61	36	0.098	0.092	0.029

The results in Table 1 were corrected for tangential cooling effects and for Liepmann and Laufer's results it was assumed, following Townsend,[8] $\langle q^2\rangle_{max} = \frac{3}{2}(\langle(u')^2\rangle_{max} + \langle(v')^2\rangle_{max})$. This gives agreement to within 4% for the author's measurements. Wygnanski attributed his lower value of σ (i.e., a faster rate of spread) to the use of a trip wire at the separating plate edge and this explanation correlates with the higher turbulence levels observed. The remaining data agree with the new constant eddy-viscosity hypothesis, i.e. $K_\gamma A = 0.094 \pm 0.002$ but the coefficient $K_G A$ varies by 15%.

Measurements of $\langle q^2\rangle_{max}/(u_1-u_2)^2$ are compared with Eq. (10) in Fig. 3 and the agreement is encouraging. The increase in $\langle q^2\rangle_{max}/(u_1-u_2)^2$ with u_2/u_1 is associated with changes in the structure of the turbulence. This has important repercussions when the problem of jet noise is under consideration and it is a phenomenon worthy of further investigation.

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Transient Gas Concentration Measurements Utilizing Laser Raman Spectroscopy

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Introduction

A NUMBER of problems in the field of gas-dynamics require sophisticated techniques for measuring the pressure, temperature, and specie concentrations of unsteady, multispecie gas flows. Such problems as the study of reacting boundary layers, gas combustion systems, and expanding jets are among the many applications of these techniques. Although methods for measuring pressure and temperature of these flows have been developed, little has been done to develop an adaptable means of measuring the concentration of one gas specie in the presence of a dissimilar gas.

Recently, the technique of applying the laser Raman effect, whereby individual gaseous species are identified by a wavelength shift in scattered light, has developed into a usable engineering diagnostic tool. The inherent problem with the Raman scattering technique is the very small Raman scattering cross section of gases. Thus, even recent work with gases has involved long-time integration to attain usable signals.[1–4] The advent of the high-power pulsed lasers, however, has allowed single-pulse Raman spectroscopy.[5–8] Widhopf and Lederman[7] demonstrated, with a single pulse ruby laser, that gaseous concentrations could be measured from a lower limit of a few torr partial pressure.

In the system described in this paper,[8] a gas is injected into a spherical chamber which contains an originally quiescent, dissimilar gas. A multiple-pulsed nitrogen laser with single-pulse

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